Quantitative description of coherent transport through surface-disordered wires

Jörg Doppler1, Otto Dietz2,3, José A. Méndez-Bermúdez4, Johannes Feist5, Florian Libisch1, Dmitry O. Krimer1, Nykolay M. Makarov6, Felix M. Izrailev4, Hans-Jürgen Stöckmann5, Ulrich Kuhl1,2 and Stefan Rotter2
1Institute for Theoretical Physics, Vienna University of Technology, Vienna, Austria, EU
2Fachbereich Physik, Philipps-Universität Marburg, Germany, EU
3Institut für Physik, Humboldt-Universität zu Berlin, Germany, EU
4Instituto de Física & Instituto de Ciencias, Benemérita Universidad Autónoma de Puebla, Puebla, Mexico
5ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, USA
6LPMC, CNRS UMR 7336, Université de Nice Sophia-Antipolis, Nice, France, EU

• Transport from mode n to n’ determined by partial attenuation length

\[ L_{\text{att}} = \sum_{n=1}^{N} \frac{1}{|\lambda_n|} \left[ W(k_n + k_{n'}) + W(k_n - k_{n'}) \right] \]

• Amplitude-gradient scattering (AGS)

\[ W(k) = \left( F_k \xi^n(x) \right)^2 \]

• Square-gradient scattering (SSG)

\[ S(k) = \left( F_k \xi^n(x)^2 \right)^2 \]

Rough boundary systems relevant in many physical systems

Optical fibres [2]
Quantitatively bound neutrons [3]
Graphene nanoribbons (4)
Silicon nanowires [5]

Surface scattering theory (SST) [6]

• Theoretical predicted new scattering mechanism


Step boundary analytically modelled by 2N+1 smeared out steps \( \Pi_n(x) \) featuring random heights \( z_n \):

\[ \eta(x) = \sum_{n=1}^{N} a_n \eta_n(x) \]

Limited wave resolution causes smearing \( \propto p \)

\[ W(k_n) = \frac{2}{\pi} \frac{4 e^{i \pi}}{\sin^2(\pi k_n \Delta / 2)} \]

\[ S(k_n) = \frac{1}{2} \frac{4 e^{i \pi}}{\sin(2 \pi k_n \Delta)} \]

\[ \Omega_n(x) = \frac{1}{2} \left( 1 + \frac{1}{2 \pi} \right) \left( 1 + 2 \cos(x) + \frac{1}{2} \sin^2 \left( \frac{\pi x}{2} \right) \right) \]

Resonance condition for specific parameter pairings:

\[ k_x \Delta = 2\pi M \quad \Rightarrow \quad W(k_x) \rightarrow 0 \quad S(k_x) \propto N \]

Numerical results

• Two open modes
• Scan through different step-widths \( \Delta \)
• Three symmetry classes:

- Symmetric wire
- Antisymmetric wire
- Nonsymmetric wire

- Standard SST [6] fails to reproduce behaviour of first mode
- Explanation: backscattering increased due to mode mixing by forward scattering
- Corrections via effective higher order scattering contributions to the attenuation length

- Direct application of higher order scattering terms:
- Quantitative agreement also in this case

Designing transmission bandgaps [8]

Summary

- Quantitative agreement between numerics and theoretically predicted new scattering mechanism
- Step-like wire geometry:
  - Pronounced backscattering peaks at resonant points \( k_x \Delta = 2\pi M \)
  - Effective higher-order scattering corrections improve agreement
- Waveguide design to fabricate predetermined transmission bandgaps
- Successful demonstration in microwave experiments and numerical simulations

References


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Contact: joerg.doppler@tuwien.ac.at

[Diagram showing waveguide design and microwave experiments, with theoretical and experimental values for AGS and SSG graphs, and SGS graphs with theory and experiment.]