Quantum signature of reconnection bifurcations

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Abstract

In this paper, we analyze quantum diffusion across a reconnecting zone in a regime of strong chaos. Quantum systems with chaotic classical counterpart show localized eigenstates even in the absence of classical regular structures. Localization can be related to invariant classical structures in phase space. In this paper we show confined quantum eigenfunctions by classical twistless curves. This feature is illustrated using a nonmonotonic standard map. © 2001 Elsevier Science B.V. All rights reserved.

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In the study of quantum chaos, many works are studied on quantum systems whose classical counterparts obey the Kolmogorov–Arnold–Moser (KAM) theorem. That is to say, changing the external or driven parameters, the invariant curves gradually break up and local chaos becomes global chaos, and the classical motion becomes diffusive. There are however, another class of systems which are out of the KAM frame. For example, those that are degenerate do not satisfy the KAM theorem. For a system with two degrees of freedom, the nondegeneracy condition corresponds to the fact that the rotation number, which determines the increase of the angle at each step of the map, varies monotonically along the invariant circles of the unperturbed map. In this case, for a pertinent resonant torus of $H_0$ ($H = H_0 + \varepsilon H_1$), Poincaré’s theorem proves in general the continuation for $\varepsilon \neq 0$ of an even number of periodic points with interchanging stability character, in agreement with the Poincaré–Birkhoff theorem for the existence of fixed points in such twist maps. Thus, in the case of twist, in the domain near a resonant torus of $H_0$, a resonant zone appears on the Poincaré section

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of the perturbed system, where the stable and unstable fixed points form a so-called Poincaré–Birkhoff chain. This bifurcation scenario, however, is very different when the rotation number attains a stationary value in an open domain. In this case, new phenomena as saddle-center bifurcations, formation of dimerized chains and vortices and heteroclinic reconnections of the asymptotics manifolds of the unstable fixed points take place [1]. Our purpose in this letter is to give numerical evidences that reconnection bifurcations exhibit fingerprints in changing the way quantum suppression of classical diffusion is reached. To show that we have studied the nonmonotonic standard map introduced by Howard and Hohs [2] in the classical and semiclassical regimes. The map reads:

$$p_{n+1} = p_n + K \sin \theta_n \quad \theta_{n+1} = \theta_n + f(p_{n+1}),$$

(1)

where $f(p) = p - \alpha p^2$. The standard nonmonotonic map (1) is integrable for $K=0$. In this case, the twist condition is violated along the integrable shearless line $p_s=1/(2\alpha)$. Also periodic orbits are created in pairs, that is, orbits with the same rotational number are symmetrically located along $p = p_s$. Using this symmetry, we define $P = p - p_s$ and the first order periodic family of -period 1 orbits- are given by $(P_m', \theta_m)$, where $\theta_m = 0, \pi$ and $P_m' = (\pm p_s \sqrt{1 - 8m\pi/\alpha})$. Our discussion will be mainly centred on the case of $m = 0$, the largest resonance in the system and we assume that the map (1) is in a regime of strong chaos for $K > 4$. The upper separatrix of the $(P_0 = -p_s, \theta_0 = \pi)$ fixed point touches the $(P_0 = p_s, \theta_0 = 0)$ fixed point of the corresponding upper chain when $x_{rec} = 1/\sqrt{12(K)}$ and it gives us a rough estimate of the reconnection $x$ value. Fig. 1 illustrates the region of the shearless curve. Any initial condition taken below the barrier will not be able to reach $p \to \infty$ even in the limit of $n \to \infty$. Fig. 1. Poincaré section for (a) $x=0.03$ and (b) $x=0.18$ to illustrate the scenario before and after reconnection respectively.
In order to compute a local diffusion coefficient for the action $p$, defined as $D = \langle (p - p_0)^2 \rangle / 2n$ over the phase-space domain of interest. In the regime of strong chaos, $\theta_i$ are random uncorrelated variables and one gets the very well known quasilinear result $4D_{QI} / K^2 = 1$ [3]. To compare the classical dynamics with its quantum counterpart, we study the quantum map written as:

$$\psi_j(n + 1) = \sum_{m = -\infty}^{\infty} U_{jm} \psi_m(n), \quad U_{jm} = e^{-i\hbar T \left( \frac{\ell^2}{2} - \frac{\hbar J}{3} \right)} J_{m-j} \left( -\frac{\omega^2 T}{\hbar} \right),$$

where the time evolution operator is written in the momentum-representation. Here $J_i$ denotes the ordinary Bessel function of the first kind. Without any loss of generality of the quantum system, we can define $\tau = \hbar T$, $\kappa = (\omega T)^2 / (\hbar T)$, and $T = 1$. It follows then $\kappa = K / \tau$, where $K = (\omega T)^2$ is the classical parameter. From these definitions, the map (1) is recovered. For the quantum model, we have numerically iterated Eq. (2) to obtain the time evolved $\{\psi_j(t)\}$, starting from various initial $\{\psi_j(0)\}$ sets. For each set, we calculated at each iteration the probability distribution $\rho(j) = |\psi_j|^2$, the average energy $\langle E \rangle = \sum_j \tau^2 (j^2 / 2) \rho(j)$, being then able to compute the quantum diffusion coefficient as $D = 4\langle E \rangle / (K^2 n)$. As it was expected, we show in Fig. 2 that the quantum diffusion follows its classical counterpart up to the Heisenberg time. After that, there is the quantum localization or suppression of the classical diffusion for both cases (a) and (b), before and after reconnection respectively. We claim that the shearless curve or the ‘barrier’ affects the way the dynamics reaches the Heisenberg time. As a consequence of the classical shearless curve, an initially localized wave packet stay localized in the region below or above the barrier as time evolves. The confined quantum eigenfunctions
Fig. 3. $|\psi(j)|^2$ for indicated times and $K = 5, \tau = 0.25$. In (a) $\omega = 0.03$ and (b) $\omega = 0.18$.

by this classical barrier is shown in the Figs. 2 and 3. In Fig. 3, $\omega = 0.07$ and one sees the signature of the barrier. However, for $\omega = 0.18$ the reconnection have occurred and the quantum probability diffuses easily over momentum space. More refined results are under way and will be soon published elsewhere.

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References